Financial Intermediation at Any Scale For Quantitative Modelling (1/3)

Cours Bachelier

Charles-Albert Lehalle

Capital Fund Management, Paris and Imperial College, London

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Motivation

- Since the 2008-2009 crisis legislators' and regulators' viewpoint on financial markets changed,
- They target to monitor and limit the risk taken by the market participants,
- In one sentence: they want to ensure most participants plays a role of **intermediaries**, and nothing more.
- The notion of intermediation and the role of banks, investment banks, dealers, brokers, and now insurance companies and funds have evolved and continue to evolve;
- important concepts to understand this are: **microstructure** and **infrastructure**; they are linked to **liquidity**.
- These last 10 years, the field of Market Microstructure emerged. Related literature has grown...
- I am convinced **financial mathematics** can address quite efficiently core concepts, as partly an academic and partly a professional, I dedicated the last 12 years to understand these changes from a practical and a theoretical viewpoint.
- These sessions will be the occasion to share how, in my opinion, financial mathematics can **answer to new and important questions** raised by recent changes.

The Standard Pitch

Following the 2008 crisis, the financial system changed a lot:

- "Clients" (from inside or outside) have no more appetite for sophisticated products.

⇒ The system went **from a bespoke market to a mass market**.

Bespoke means to sell products that are very different: no economies of scale but high margins.

Mass market means a lot of similar products + optimized logistics.

- Regulators welcome this change because it can prevent an accumulation of risk in inventories (cf. optimized logistics).

⇒ The G20 of Pittsburgh (Sept. 2009) put the emphasis on **inventory control** (it is the root of improved clearing, segregated risk limits, etc).

⇒ Policy makers took profit of two existing regulations (Reg NMS in the US and MiFID in Europe) to push toward **electronification** of exchanges (i.e. improved traceability and less information asymmetry).

- Technology went into the game. Think about the kind of recent "innovations" (uber, booking.com, M-pesa, blockchain, etc): it is about **disintermediation**.

⇒ How do you desintermediate a system made of intermediates?

Historically, market **micro**structure stands for not reducing

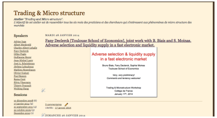
- Sellers = Equity Shares and Bonds issuers
- Buyers = investors.

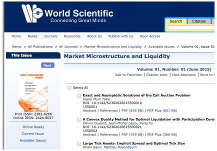
In practice, today, associated topics are

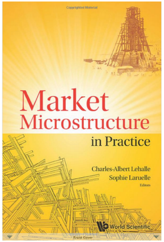
- Market impact, Fire sales and Flash Crashes
- Auction / Matching mechanisms (Limit Orderbooks, RFQ, conditional / fuzzy matching, etc)
- Optimal trading / Liquidation
- Market Making and High Frequency Trading
- Investment process while taking all this into account

My Viewpoint on Market Microstructure

I have been Global Head of Quantitative Research at Crédit Agricole Cheuvreux and CIB during years (including the crisis). I discuss a lot with regulators; previously inside the **working group on Financial Innovation** of the ESMA, now inside the **Scientific Committee** of the AMF. I am now in a large Hedge Fund.







- From a **Financial Mathematics** perspective, it is nothing more than adding a variable to our models: the **Liquidity** .
- The interactions between liquidity and other (usual) variables is far from trivial.

Disclaimer: I express *my own opinion* and not the one of any of these institutions.

What Will We Do During 3 Lectures + 1 Seminar?

I will not go in the details of the models (except for few of them), because I target to give you enough information to include liquidity in the models you know better than me.

Hence, I will

- 18 Nov:
  - Start by the definition of **intermediation**
  - Focus on the two main **Liquidity** variables on financial market: inventories and flows
- 25 Nov:
  - Show you what **Liquidity** looks like when we can observe it
- 2 Dec:
  - Underline why **market making** (inventory keeping) and **optimal trading** (flow management) are core for the new role of market participants.
- 2 Dec [Seminar]:
  - Explain what practitioners are doing.

It is an on-going work

My own viewpoint on optimal trading:

- We have sophisticated (but tractable) methods to **optimize the strategy of one agent** (investment bank, trader, asset manager, etc) facing a “background noise” (stochastic control is now really mature),
- These methods are **used by practitioners** (already three books on this topic [Lehalle et al., 2013], [Cartea et al., 2015], [Guéant, 2016]),
- Differential games, and more specifically **mean field games** now propose very promising frameworks to replace most of the background noise by a mean field of explicitly modelled agents:
  - to provide robust results for practitioners [Cardaliaguet and Lehalle, 2016],
  - to obtain meaningful results for policy recommandations [Lachapelle et al., 2016].

Up to now most results on global modelling used a simplification of a reality. Now decisions are modelled and systematic, why not inject them into a global model?
It should enable you to produce very accurate models and draw powerful conclusions.

- Beyond optimal trading, these lectures should help you in introducing liquidity in any model of yours: **please ask question!**

Outline of the Sessions

Risk Transformation 101

To understand the interactions between actors of financial markets, a first step is to understand the **role of the financial system**.

It takes its role at the root of capitalism:

- say you see a shoes shiner at Deli, India
- you pay $1 to have your shoes shined, and you ask to the guy
- "it seems you have around 30 customers each day, it let you with $30 every day, it is a good job."
- he answers: "not at all, I earn $1 a day... I do not own the brush, its owner loans its to me $29 a day. Since a brush costs $12 and I need my daily dollar to eat, I will never own one."

→ let's discuss about microcredit: loan him $12 during 2 days...

You have $30, you can ask to the guy some percents to cover the risk he will not have enough clients. If you are risk averse, you can even ask for the brush as collateral... A bank can "structures" the loan for you, it will take care of all the administrative aspects, it is a simple **risk transformation** (liquidity on you side, business of the shoes shiner side).

If you want to implement microcredit on your own, it is impossible: **you will never cross the path of someone in the situation of the shoes shiner**. The bank can find borrowers and lenders. As an intermediary, it should do two thinks

- concentrate the flow, building a **marketplace** ;
- provide neutral **information** to both sides of the loan;
- take care of **collateral** ;

it deserves fees that for.

The bank can even have an incentive to **make the market** : if a borrower is there but not lender is present, it can provide the loan itself, and wait for the next lender.

$\Rightarrow$ how can you make the difference between a bank waiting for the next three lenders and a bank taking **directional risk** itself?

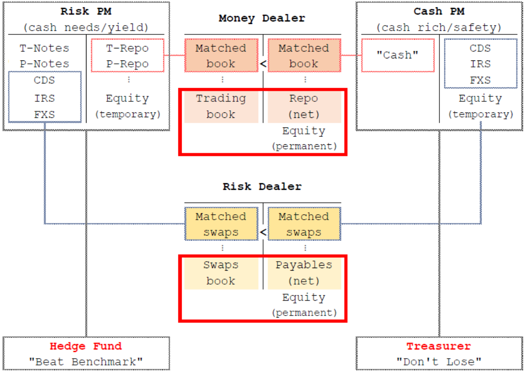
The Subtle Balance of Intermediaries' Inventories

If we take the viewpoint of looking at the financial network **from the outside** , we need to understands its inputs and outputs, and deduce the features it provides to the rest of the economy.

- We can see how the banking network operates a maturity transformation between natural borrowers at different maturities (mid or long term) and natural lenders (short or mid term).
- Investment banks operate the same way with a lot of other risk transformation (insurance –ie optional payoffs–, structured products, swaps, etc),
- Banks are intermediaries: they have no reason to keep risk in their inventories.
- The bad cases are when all the banks host risks in the same direction (2008), instead of having a diversification at the scale of the whole system.
- Banks are often tempted to take directional risk (sometimes without really knowing it). It is the goal of regulators to force them to maintain the risk of their inventory as low as possible (using capital requirements).
- Nevertheless inflows and outflows in Banks balance sheets (i.e. *transactions*) are not simultaneous, hence regulators need to give them some freedom to wait for a seller once they sold a contract to a buyer (and the reverse).

Let's see this viewpoint is typically a microstructural one: intermediaries, buyers and sellers, inventory risk... Most of the buzz words are there...

More Sophisticated: Risk Trading *Inside* the System



Source: Pozsar on Shadow Banking (2013)

On the one side (right) you have **Cash PMs** they are *cash rich* but *safety poor* (“fear to loose their money”).

On the other side (left) you have **Risk PMs**, they have to beat a benchmark, thus are *securities rich* but *return poor* (the need leverage and non linearities).

In between (middle) you have **intermediaries**, they match Risk PMs on the asset side of their balance sheet and Cash PMs on the liability side.

This is accountancy, each time a transaction of this kind is made, it has to be marked- to-market, thus all this is pegged to traded prices.

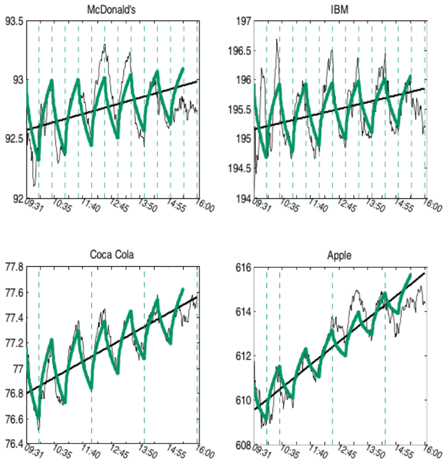
You need such mechanisms to spread the risks across the system: with chance the net exposure of one intermediary will meet the opposite hosted by another intermediary. You will **net risk inside the system** (Clearing is crucial).

Going back to concepts mathematical finance is more familiar with:

- you are an investment bank, you sell a structured product or a derivative to clients;
- you do not hedge each book separately (or at least you shouldn't): you hope to have other clients consuming other products flattening your (risk) inventory.
- Of course you will not succeed in netting 100% of the risk, hence **you have to hedge the remaining book**, in the markets (we hope they use optimal trading algorithms –i.e. continuous trading– to do this).
- But one step further: if you succeed into hedging continuously on markets (without liquidity, i.e. market impact, issues), it just mean someone has the opposite risk in the market and hedges it on its side: you should / could find it and net both positions (think about the crucial role of CCP here).
- In this sense **wrong way risk is not good for the liquidity on markets** at all, you cannot believe you really hedge if you impact the price.

Two good (but stylized) examples in the literature are [Stoikov and Saglam, 2009] and [Carmona and Webster, 2012].

FIGURE 1: SAWTOOTH PATTERNS ON COCA-COLA, MCDONALD'S, IBM AND APPLE ON 19 JULY 2012



Even on liquid stocks and for vanilla options (close to maturity in this case), hedging can go wrong.

The 19th of July 2012, a trading algorithms bought and sold shares every 30 minutes without any views on its market impact [Lehalle et al., 2012].

For one visible mistake like this on liquid underlyings of vanilla products, how many bad sophisticated hedging processes on less liquid (even OTC) markets...
Anonymous continuous hedging of a remaining position outside of the bank does not mean all is going well.
Nevertheless we have solutions in recent literature: [Guéant and Pu, 2013], [Li and Almgren, 2014].

But nothing more generic, for instance the whole process of hedging books in presence of wrong way risk is not studied (as far as I know). One step in this direction is [Schied and Zhang, 2013].

My advices to an investment bank:

- **Net all your books**, maintain two opposite positions is costly and risky,
- If you can't it may be because you do not communicate enough internally (sometimes because of Chinese walls...), hence **be ready to hedge on the market**,
- But **before try to match your small metaorders**: send them to an internal place and cross them as much as possible;
- You will have synchronization issues (at the level of these metaorders, no reason to be synchronized), ask to your traders to **implement facilitation-like market making** schemes inside the bank.
- The remaining quantity has to be sent to markets as smoothly as possible, but it does not mean you will have no impact. **Who is your counterpart in the market** should be an obsession: if you trade a one way risk, you will pay for this in the future...

The Crucial Role of Intermediaries

The are in charge of

- anonymize the orders,
- find counterparts,
- provide unconflicted advices on the value of tradable instruments.

Investment banks have

- Flow desks,
- Market making desks,
- Execution and market access services,
- Financial analysis services.

Replication and hedging is a way to minimize the risk of intermediaries' books. Ideally they should prefer to find a matching counterpart. Their efficiency deeply relies on their capability to net their positions to zero (i.e. to find buyers once the sold, and the reverse). Most often residual risk will remain in their book, and they will have to hedge it anyway.

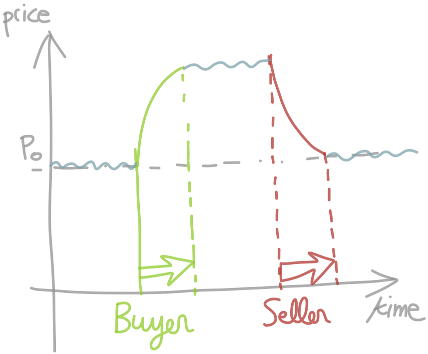
Informational Role of intermediaries

As a regulator, you need intermediaries to:

- net and secure the positions,
- ensure a fair access to (fundamental) information,
- facilitate liquidity and make the markets.

About Financial Information:

- 40 years ago, central banks' decisions were not easy to access
- 20 years ago, you needed to by physically present during CEO/CFO speeches to have accurate information about firms,
- 10 years ago, sector-wide information was not easy to access
- today, cross-asset information is difficult to access.



- On an exchange or over the counter, if buyers and seller are desynchronized, the price will not be efficient.
- **Market Makers** will “make the market” in selling to buyers at $P_t + \psi$ and buying to sellers at $P_t - \psi$
- they earn the bid-ask spread ψ and take an **inventory risk**.

Market Makers Problematic

They earn the bid-ask spread and take an **adverse selection risk**: what if the price is really changing? They will never buy-back at a good price...

Intermediation is about providing unconflicted information, anonymity, price dissemination and concentrate the trading flow (see [Scholtens and van Wensveen, 2008] and [Merton, 1995]).

Remark 1 (*The Process of Intermediation of Liquidity*)

If buyers and sellers of an instrument are not synchronized, intermediaries will answer to the first, take the risk in their book, and wait for the other.

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If buyers and sellers of an instrument are not synchronized, intermediaries will answer to the first, take the risk in their book, and wait for the other.

If the price does not move that much and buyer and sellers come according to a Poisson process, things should be ok. Nevertheless intermediaries' books face a risk (what if the price move between two arrivals?).

New Matching Technologies? If you want to go further: you can think about a world in which all the matchings were automated, taking places on electronic venues (i.e. *trading facilities*). Even sophisticated products could be matched electronically.
Just have a look at `ripple` and `blockchain` (the protocols underlying the Bitcoin crypto-currency), or `ethereum.org` (another protocol dedicated to complex contracts).

When a market maker says “yes” to every sollicitation...

Uncontrolled Inventory of a Market Maker (MM)

If the arrival of buyers (resp. sellers) follows a Poisson process with intensity λ^- (resp. λ^-), when a MM is continuously present at the best bid and ask, its inventory I asymptotically follows the Brownian motion

$$d\mathcal{I} = (\lambda^+ - \lambda^-)dt + \sqrt{\lambda^+ + \lambda^-} \cdot dW_t. \tag{1}$$

It is enough to note the dynamics of inventory $\mathcal{I}$ are $d\mathcal{I} = dN^+ - dN^-$ and to refer to the construction of Brownian motion using a Poisson process. $\square$

Hence a MM has to ask herself if $\lambda^+ = \lambda^-$? i.e. is my client flow unbalanced ? do they have (private) information?

Moreover, she even cannot afford to have a diffusive inventory (i.e. the dW part), hence she will **control** its inventory using a 2D parameter (δ^+, δ^-) to obtained as dynamics:

$$d\mathcal{I} = dN^+(\delta^+) - dN^-(\delta^-).$$

$\rightarrow$ what could be (δ^+, δ^-) ?

Controlling the Risk

- Regulators want the market makers to control the risk they accumulate in the system.
- But they need to allow them enough inventory to wait for the next participant,
- Otherwise the market will freeze

The Market Maker Controls

The MM wants to decrease λ^+ and increase λ^- (resp. λ^-/λ^+) when her inventory is positive (resp. negative). When it is possible to use (δ^+, δ^-) so that $\lambda^+(\delta^+) - \lambda^-(\delta^-) \propto -I$, then the market maker inventory dynamics are Ornstein-Uhlenbeck:

$$d\mathcal{I} = -\gamma I\, dt + \sigma dW,$$

and hence asymptotically $\mathcal{I} \sim \mathcal{N}(0, \sigma^2/(2\gamma))$.

This a way to control the risk of a market making strategy. Several papers have been written on this

- *Optimal dealer pricing under transactions and return uncertainty* – [Ho and Stoll, 1981] (economics)
- *High-frequency trading in a limit order book* – [Avellaneda and Stoikov, 2008] (first quantitative model)
- *Dealing with the inventory risk: a solution to the market making problem* – [Guéant et al., 2013] (full solution)
- *Market Impacts and the Life Cycle of Investors Orders* – [Bacry et al., 2015] (toy model using Hawkes processes)
- see Olivier Guéant's book [Guéant, 2016] for details.

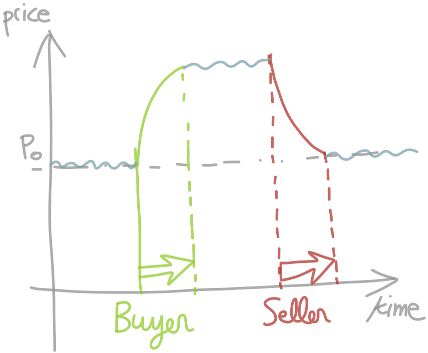
Regulators do not want to let intermediary **take too much risk** because it is very difficult to check what they have in it.
Assume this toy model: buy and sell orders intensity are the same $\lambda^- = \lambda^+ =: \lambda$, then the inventory of a market maker (MM) accepting all trades evolves like

$$d\mathcal{I} = \underbrace{\sqrt{2}\lambda}_{\sigma} dW.$$

If a regulator set a "risk limit" at $A_q\,\sigma$ (say it is the qth quantile of a Gaussian: $A_q := \Phi(q)$), then as soon as $|\mathcal{I}| > A_q\,\sigma$, the MM stops to accept trades in one of the two directions, she has to wait on average λ units of time.

Freezing Market Makers

In our toy model: when the regulator set the *risk limit* at $R := A_q\,\sqrt{2}\lambda$ the market "freezes on average" as soon as $\Phi^{-1}\left(R/(\sqrt{2}\lambda)\right) > \lambda$.



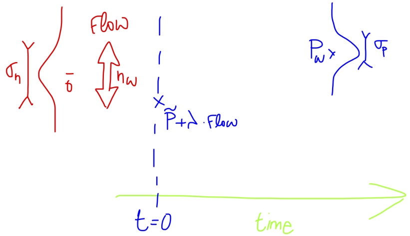
As we just have seen

- Adverse selection is the main issue a market maker should face.
- On the other side, the liquidity taker will face opportunity cost: you buy now, but few minutes later, the price would have been better.

If the market maker only deals with someone having directional information for 100% of the trades, the market maker will bankrupt. Market makers do not usually invest in extracting “fundamental” (i.e. exogenous) information. Nevertheless

- Investment banks have analysts and multiple clients,
- Systematic market makers have newsfeeds.

Making the Market in Front of Adverse Selection: Kyle's 85 Model



The framework

- An informed trader, knowing the future price
- Noise traders, knowing nothing
- A market makers, having only access to distributions (thanks to "backtests" / observations); she changes her price linearly according to the price pressure she observes: $f_P(q) = \tilde{P} + \lambda \cdot q$.

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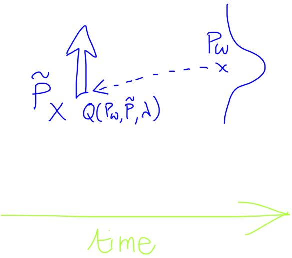

The informed trader chooses his quantity to maximize his expected profit

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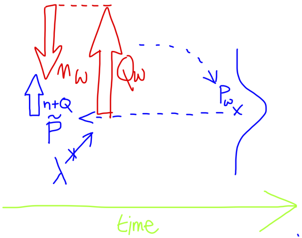

The market maker choose $\tilde{P}$ and λ to adjust her price to the flow

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- The informed trader adjusts his participation to maximize its profit (given $\tilde{P}$ and λ),
- The market makers know the distribution of the informed price and set $\tilde{P}$ and λ so that her price is as close as possible to its expectation.

Following *Continuous Auctions and Insider Trading* – [Kyle, 1985]:

- Remember the market makers fear adverse selection.
- We have **informed traders**, they know the price will be p_ω after their trade, $p_\omega \sim \mathcal{N}(P^*, \sigma_p^2)$.
- Other traders, (i.e. *noise traders* for Kyle) trade for other reasons, their net direction is $n_\omega \sim \mathcal{N}(0, \sigma_n^2)$.
- The informed traders have to **choose a participation** $Q(p)$ (they know p) to maximize their profit,
- Knowing the market makers (MM) will react to the net perceived flow **linearly**: the *public price* will be

$$f_P(Q(p_\omega) + n_\omega) = \tilde{P} + \lambda \cdot (Q(p_\omega) + n_\omega).$$

- Moreover in their filtration, the MM should produce a price being the **best estimator of** p_ω **given** $Q(p_\omega) + n_\omega$.

- **Informed traders** maximize their expected price: $\arg\max_Q \mathbb{E}((p_\omega - f_p(Q + n_\omega))Q|p_\omega)$.

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The Economics of Market Impact: Kyle's 85 Solution

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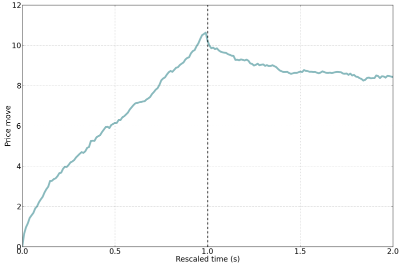
$$\left\{ \begin{array}{lcl} P^* & = & \tilde{P} + \lambda \mathbb{E}(Q(p_\omega) + n_\omega) \\ \lambda & = & \dfrac{\mathbb{C}\mathrm{ov}(p, Q(p) + n_\omega)}{\mathbb{V}(Q(p) + n_\omega)} \end{array} \right. \Rightarrow \left\{ \begin{array}{lcl} \tilde{P} & = & P^* \\ \lambda & = & \dfrac{\sigma_p^2/(2\lambda)}{\sigma_p^2/(2\lambda)^2 + \sigma_n^2} \end{array} \right.$$

$\Rightarrow$ $\boxed{\text{It can be solved with } \lambda = \sigma_p/(2\sigma_n).}$

- The more potential informational price move (i.e. large σ_p), the largest impact.
- The more non informative flow, the more difficult for the MM to identify information, hence the less she impact the price.

On our database of 300,000 large orders



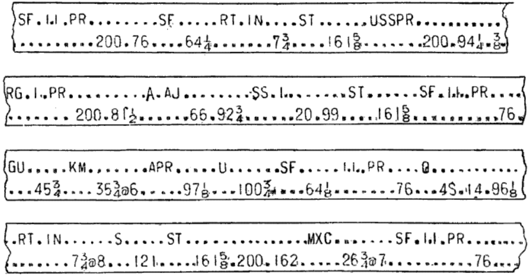
Market Impact takes place in different phases

- the **transient impact**, concave in time,
- reaches its maximum, the **temporary impact**, at the end of the metaorder,
- then it **decays**,
- up to a stationary level; the price moved by a **permanent** shift.

In [Bacry et al., 2015] we studied all the phases, using intraday and daily analysis (for the first time). We underlined the importance of some "normalization variables": the uncertainty on the price formation process , the capability of the orderbook to resist to volume pressure , and the duration of the metaorder.
Following [Waelbroeck and Gomes, 2013] and simultaneously with [Brokmann et al., 2014], we proposed an explanation of **permanent impact** .

Ticker Machine

Ticker Tape

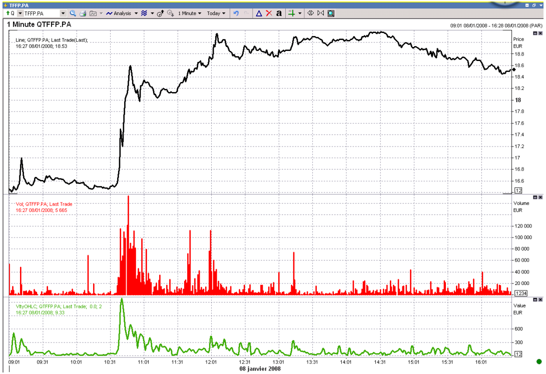


The stock "tape" as it comes from the "ticker."

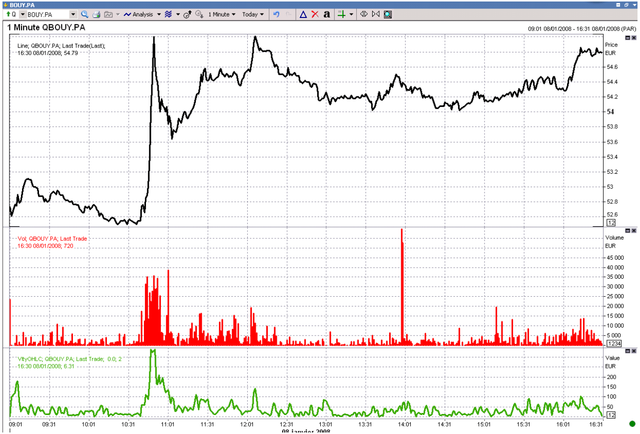
The price is not only the outcome of the balance between liquidity providers and liquidity consumers (not really between buyers and sellers from our perspective). It is an **input** for most agents decision processes. They all pay a lot of attention to the price (most of the positions have to be **marked to market**). The price of a transaction has more importance than any indicative one.

In theory market makers provide liquidity and fear adverse selection. Their protection is to **impact the price** (i.e. to statistically guess how much information is in the flow).

We can select very specific dates with exogenous information or flow and look at the prices and volumes.

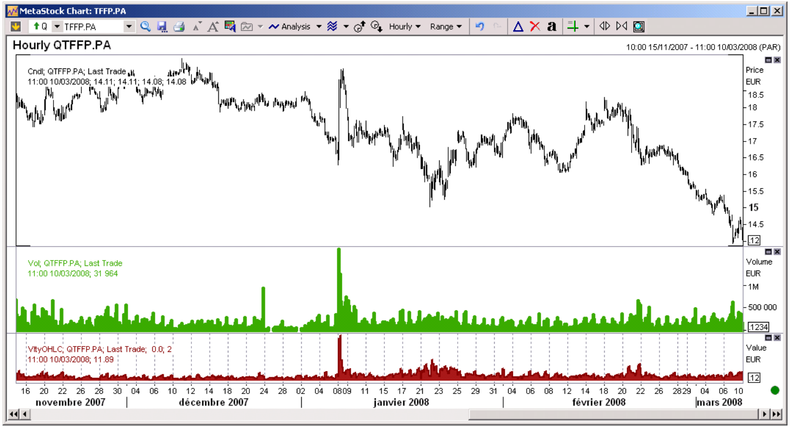


This is the effect of the unexpected announcement by president Sarkozy (France, 2008) |that public TV channels will no more be allowed to sell advertising... The value of the main french private Channels jumped immediately.

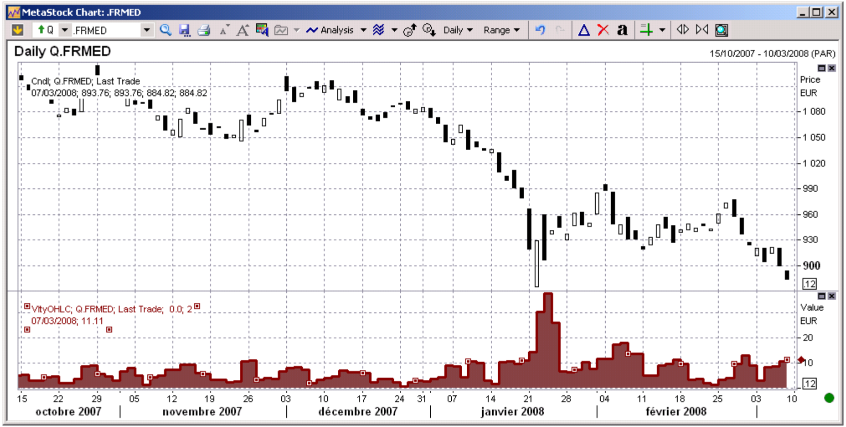


Bouygues price around the announcement of the end of advertising on public channels (8 Jan. 2008).

Propagation - zooming out: 30 minutes


MetaStock Chart: TFFP.PA
TFFP.PA
Analysis
Hourly
Range
Hourly QTFFP.PA
10:00 15/11/2007 - 11:00 10/03/2008 (PAR)
Cndl; QTFFP.PA; Last Trade
11:00 10/03/2008; 14.11; 14.11; 14.08; 14.08
Price
EUR
18.5
18
17.5
17
16.5
16
15.5
15
14.5
12
Vol; QTFFP.PA; Last Trade
11:00 10/03/2008; 31 964
Volume
EUR
1M
500 000
1234
VltyOHLC; QTFFP.PA; Last Trade; 0.0; 2
11:00 10/03/2008; 11.89
Value
EUR
12
novembre 2007
décembre 2007
janvier 2008
février 2008
mars 2008


MetaStock Chart: TFFP.PA
Daily QTFFP.PA
15/10/2007 - 10/03/2008 (PAR)
Cndl; QTFFP.PA; Last Trade
10/03/2008; 14.43; 14.43; 14.05; 14.09
Price
EUR
20
19
18
17
16
15
Vol; QTFFP.PA; Last Trade
10/03/2008; 198 311
Volume
EUR
4M
2M
VltyOHLC; QTFFP.PA; Last Trade; 0.0; 2
10/03/2008; 53.84
Value
EUR
15 22 29 05 12 19 26 03 10 17 24 31 07 14 21 28 04 11 18 25 03 10
octobre 2007
novembre 2007
décembre 2007
janvier 2008
février 2008
mars 08


MetaStock Chart: .FRMED
Daily Q.FRMED
15/10/2007 - 10/03/2008 (PAR)
Cndl; Q.FRMED; Last Trade
07/03/2008; 893.76; 893.76; 884.82; 884.82
Price
EUR
1 080
1 050
1 020
990
960
930
900
VltyOHLC; Q.FRMED; Last Trade; 0.0; 2
07/03/2008; 11.11
Value
EUR
20
10
15 22 29 05 12 19 26 03 10 17 24 31 07 14 21 28 04 11 18 25 03 10
octobre 2007
novembre 2007
décembre 2007
janvier 2008
février 2008

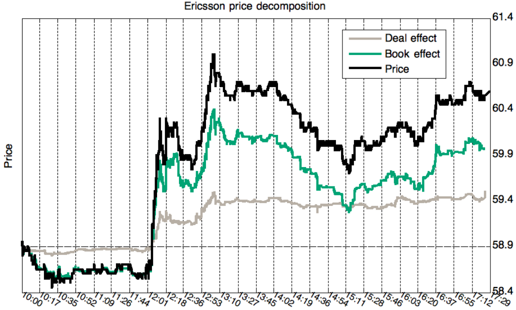
Effets of Such an Unexpected Information

- The price changed almost immediately (up), with large volumes, but oscillated a little bit
- Other (related, correlated) stocks, moved accordingly
- Sectorial moves drove the prices down.

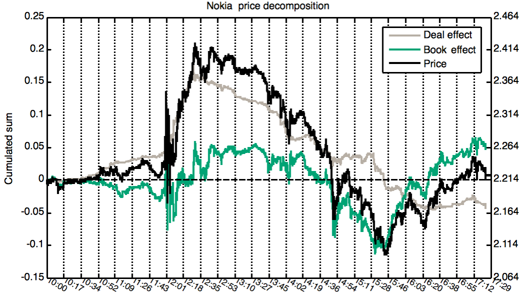
We saw a jump that can be explained at 100% by an exogenous (i.e. "fundamental") information. We could say **information drove prices** . In such a case there is no proof of causality like: buying pressure $\rightarrow$ price move. It is more a **common agreement** on a new price.

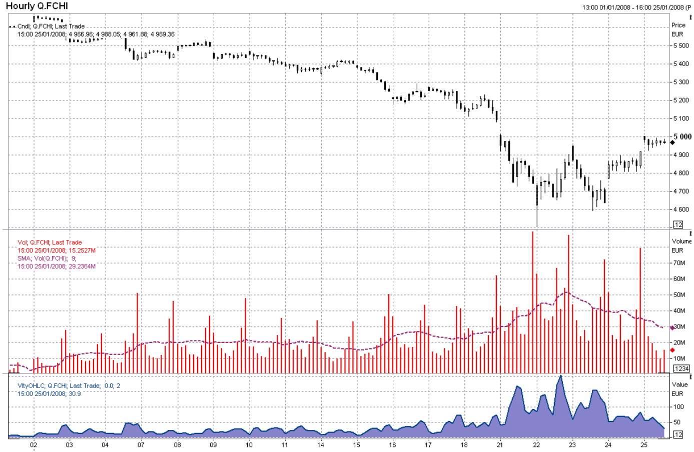
In [Besson and Lehalle, 2014], we try to split price moves caused by trades or just “quote shifts”, we provided aggregated statistics but here are two anecdotical cases, the 18th of Oct. 2012; it shows how news impact the price, it somehow shows participants have several ways to impulse permanent impact:

easy to process news



difficult to process news





Market impact generated by the unwind of Jerome Kerviel's position

We Have Seen Few Different Cases

TF1 — “Pure” information : the causality is information $\rightarrow$ prices $\rightarrow$ volumes $\rightarrow$ oscillations,

Ericsson — Information, easy to process : prices did not needed transactions to move,

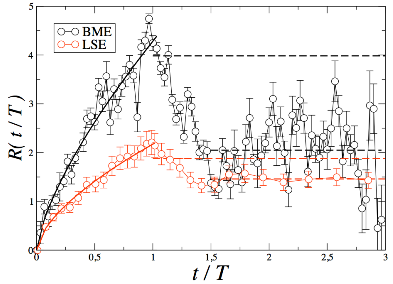
Nokia — Information, but difficult to process : prices moved because of transactions pressure,

Kerviel — No information (i.e. “*Cash Trades*”) the causality is for sure volumes $\rightarrow$ prices.

Where does the “permanent impact” comes from? Is it

- Price discovery ? price “had to goe there”,
- or Price Formation ? mechanical pressure of the traded volume on the prices.

in [Moro et al., 2009]



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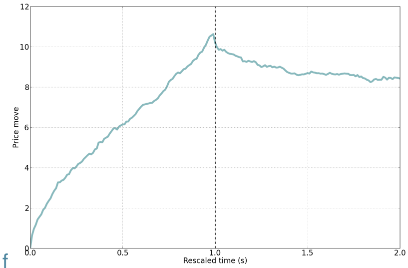

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To be more than anecdotical, it is needed to make statistics, that for we need a not of occurrences of "**metaorders**". Some paper documented the "**square root impact**": the temporary impact of your flow is proportional to the its square root. But the three phases has been studied in fewer papers:

- *The Non-Linear Market Impact of Large Trades: Evidence from Buy-Side Order Flow* [Bershova and Rakhlin, 2013] – intraday impact
- *Is Market Impact a Measure of the Information Value of Trades? Market Response to Liquidity vs. Informed Trades* [Waelbroeck and Gomes, 2013] – daily impact of cash trades
- *Slow decay of impact in equity markets* [Brokmann et al., 2014] – daily impact of informed trades (hedge fund)
- *Market Impacts and the Life Cycle of Investors Orders* [Bacry et al., 2015] – intraday and daily impact of informed trades (bank)

Regression	Parameter	Coef. (log-log)	Coef. (L2)	Coef. (L1)
(R.1)				
	Daily participation	0.54	0.45	0.40
(R.2)				
	Daily participation	0.59	0.54	0.59
	Duration	−0.23	−0.35	−0.23
(R.3)				
	Daily participation	0.44	–	–
	Bid-ask spread	0.28	–	–
(R.4)				
	Daily participation	0.53	–	–
	Volatility	0.96	–	–
(R′.1)				
	Trading rate	0.43	0.33	0.43
(R′.2)				
	Trading rate	0.37	0.56	0.45
	Duration	0.15	0.24	0.23
(R′.3)				
	Trading rate	0.32	–	–
	Bid-ask spread	0.57	–	–
(R′.4)				
	Trading rate	0.32	–	–
	Volatility	0.88	–	–

Source: [Bacry et al., 2015]

The Formula should be close to

$$\mathrm{MI} \propto \sigma \cdot \sqrt{\frac{\text{Traded volume}}{\text{Daily volume}}} \cdot T^{-0.2}$$

The term in duration is very difficult to estimate because you have a lot of conditioning everywhere:

- did you trading process reacted to market conditions?
- are you alone?
- etc.

We used different methods.

We had enough data to investigate long term impact, exploring the relationships between permanent impact and traded information.

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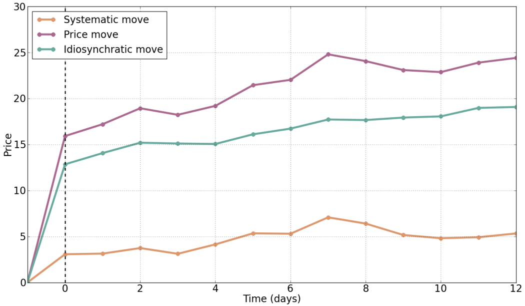
Daily price moves



- If you plot the long term price moves (x-axis in days), you see an regular increase;
- But the same stock is traded today, tomorrow, the day after, etc.

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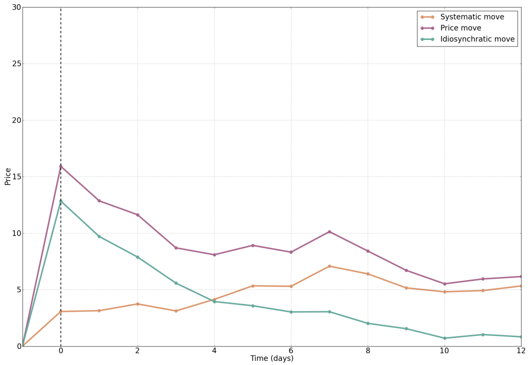


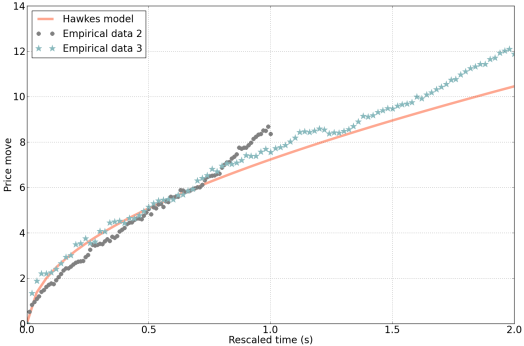
- If you plot the long term price moves (x-axis in days), you see an regular increase;
- But the same stock is traded today, tomorrow, the day after, etc.
- Once you remove the market impact of “future” trades (similarly to [Waelbroeck and Gomes, 2013]), you obtain a different shape.
- If you look each curve: the yellow one contains the CAPM β (the metaorders are trading market-wide moves), the green curve contains the idiosyncratic moves, this shape can be read as **the daily decay of metaorders impact**.

Remark 2

The main source of persistence of price move after the market impact (i.e. trades of large orders) is informational:

- *"factors" for slow, beta-driven investors,*
- *alpha for hedge funds.*

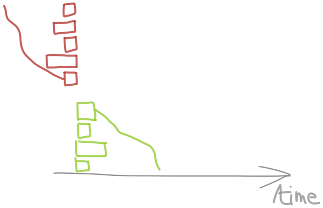
Keep in mind the difference between TF1 jump (informational) and Kerviel's inventory unwinding (mechanical).



We only have conjectures, but we know (at left) liquidity providers are not “telepathic” (same impact for the first MM minutes regardless to the duration of the metaorder). Nevertheless the concavity

- could come from “front running”,
- could come from “auto selection of market makers”,
- can be explained by an Hawkes process.

See [Bacry et al., 2015] for details.



At the highest frequency, it is possible to define the **Price Impact** (the price move just after a trade, cf. [Wyart et al., 2008] and [Gatheral, 2010]) as:

$$\Delta\tilde{P}_0(t) := \mathbb{E}(\epsilon_0(P(t) - P(0))|\text{trade at } 0).$$

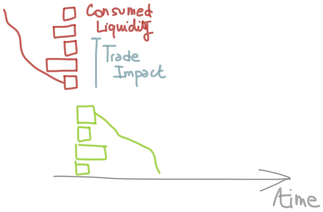
The Propagator Model

$\Delta\tilde{P}_0(t) = \eta(Q_0)K(t-0)$ where $K(\cdot)$ is a *Kernel*, usually taken as

- an exponential kernel: $K(s) = \lambda \exp -(s/\lambda)$;
- a power law: $K(s) = (1+s)^{-\gamma}/(\gamma - 1)$.

Autoregressive models can be use as propagators.

Price Impact and the Propagator Model



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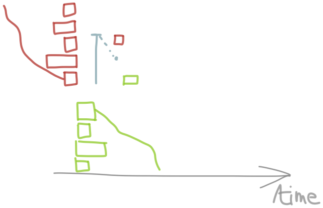

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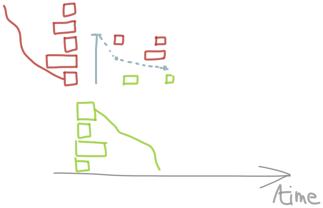

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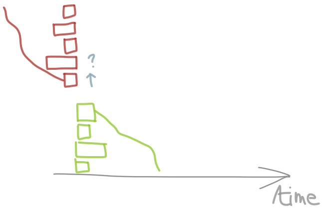

Adverse selection is the fact you obtained liquidity now and you could obtain it later and have a better price.

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Price Impact and Adverse Selection

Price Impact for market orders implies Adverse selection for limit orders.

If the price in martingale after a price change, there is adverse selection ; the imbalance says you have a little less adverse selection than that since **once a full tick has been consumed, they are chances the discovered quantity is larger than average**.
When you owns a limit order in the book: **the more orders behind you, the more protected vs. adverse selection**.

2nd worst kept secret of HFT : if you have too few orders behind you, cancel your limit order.

“discovered quantity” means the quantity at second limit that is now a first limit

When an order is sent aggressively to the market, as far as it is higher than the quantity available at the first limit in the market:

- the price changes immediately (most studies have shown that is follows a power law [Wyart et al., 2008]),
- the market provides *decay*.

This is clearly stated into Jim Gatheral's paper [Gatheral, 2010] this way: given that an order of size v hit aggressively the market at time τ, its remaining impact on the market at any t greater than τ is:

$$\delta_t(v,\tau) = \eta_\tau(v)\, G(t-\tau) \tag{2}$$

where η_t is the immediate market impact function and G a *decay kernel*.
Consequently, the cumulated effect of all trades (τ, v_τ) that occured before t is:

$$\Delta_t(v,\tau) = \int_{\tau<t} \eta_\tau(v_\tau)\, G(t-\tau)\, d\eta(\tau, v_\tau)$$

The decay comes from liquidity providers that agree to insert orders between the actual price impacted by the order of size δv and the previous price.

This can be modelled using a *fair value* for the price S_t^f. The decay is then a functional of difference between the market price S_t and this fair price:

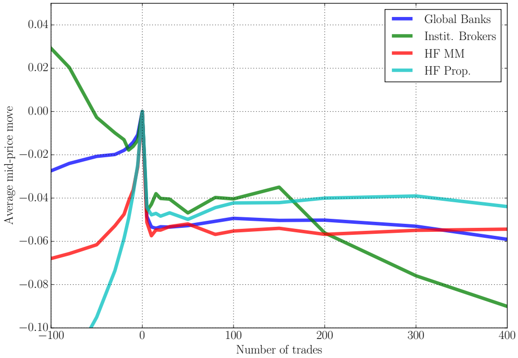
$$\delta_t(v,\tau) = \eta_\tau(v_\tau)\, G(t-\tau, |S_t - S_t^f|)$$

given that an order of size δv_τ has been sent aggressively to the market at τ for a lot of τ in the past, and if the *fair price* is modelled by a stochastic process S_t^f, we have:

$$S_t = S_0 + \int_{\tau<t} \eta_\tau(v_\tau)\, G(t-\tau, |S_t - S_t^f|)\, d\eta(\tau, v_\tau) \tag{3}$$

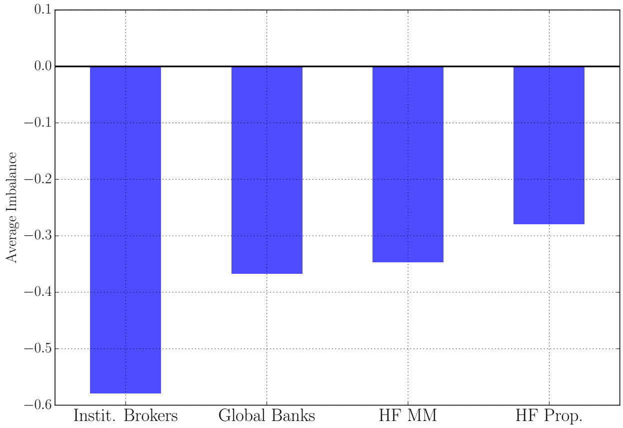
Effect of passive orders

- On average and market wide, there is no reason market orders and limit orders have a different impact on the price
- If you consider there is one limit order for one market order,
- We will see (orderbook dynamics) that is a little more complex.
- You have to consider the whole trajectories of orders to understand the real cost of a market or a limit order.
- At this stage we can just look at static pictures, conditioned by a given stopping time and the owner of the order.



Price profiles: future and past of the mid-price (solid) or bid and asks (dotted), conditionally to an execution.

- Instutional Brokers (i.e. essentially “client flows”: with a decision taken at a daily scale and large metaorders)
- HFT, split in HF market makers and HF proprietary traders
- Global banks, having a mix of client flows and proprietary trading flows.
- **Conditionally to the owner of the order**, the profile can be very different
- You can compare such graphs and make matrices [Brogaard et al., 2012]
- Nevertheless the big picture is dynamic...



Imbalance profiles: State of the book conditionally to an execution, renormalized such as best opposite is 1, the green bar is your order size.

If someone trade at a given frequency $1/\delta t$ from 0, his price impact at $K\delta t$ will be (for an exponential kernel)

$$P(K\delta t) - P(0) = \sum_{k \leq K} \eta(1)\lambda e^{-k\delta t\,\lambda} \simeq \eta(1)(1 - e^{-K\delta t\,\lambda})/\delta t.$$

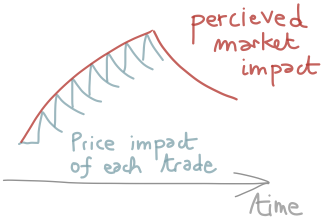
And for a power law

$$P(K\delta t) - P(0) = \eta(1)\left(1 - (1 + K\delta t)^{-(\gamma-1)}\right)/\delta t.$$

In both cases, if he stops trading at $K\delta t$, the price will **fully** revert according to an exponential (or a power lax).

Transient Impact and Decay

The concave increase of the impact with time and its reversion can be explained using propagator models.



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Common sense for Temporary Market Impact model

- Crossing the Bid-Ask spread. If the last trade was on the opposite side to my order (once over two trades...), I pay the spread: 2ψ.
- Dynamical market depth. The more the volatility of the price diffusion is high, the more the LOB is structurally empty: M.I. increases with σ.

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- Dynamical market depth. The more the volatility of the price diffusion is high, the more the LOB is structurally empty: M.I. increases with σ.
- Usual quantities: the more my volume v is large compared to the "*usual*" traded volume $V_{\Delta T}$ (during this time interval ΔT), the more my M.I. increases.

With ψ the half spread, σ the volatility (in currency), and V a constant homogeneous with a quantity of shares, The most common model is:

$$\eta_t(v) = \alpha\psi_t + \kappa\sigma_t \cdot \left(\frac{v}{V_t}\right)^{\gamma} \tag{4}$$

Usually you take $\gamma = 1/2$, κ is a function of your dataset (it corrects the denominator) and α is a function of your trading signals.

Let $f^-_{\mathrm{LOB}}(s)$ be the volume available in the LOB by sellers (ask prices) at price s (resp. $f^+_{\mathrm{LOB}}(s)$ for bids quantities). When I want to buy a quantity V of shares of this instrument, I will generate a Market Impact of S_{MI} implicitly defined by the equation (S_0 is the last quoted price):

$$V = \int_{s=S_0}^{S_{MI}} f^-_{\mathrm{LOB}}(s)\, ds. \tag{5}$$

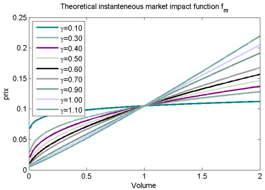
S_{MI} of (5) is the market Impact $\eta(V)$ associated to the volume V.

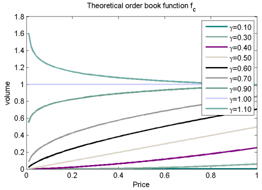
[

Property: Implicit Orderbook] Formal derivation of the market impact equation (5) with respect to V gives the *Implicit Orderbook* associated to the Market Impact model η:

$$f^-_{\mathrm{LOB}}(S_{PMI}(v)) = \frac{1}{\partial_v \eta(v)} \tag{6}$$

Tell me your price impact function $S_{MI}(v)$, I will tell you your implicit orderbook function. So if you think that market impact is constant, it means that you think that LOB is infinite.





For the generic power law market impact, we obtain:

$$f_c(S_0 + h) = \frac{V}{\kappa\sigma\gamma \left(\frac{h-\psi}{\kappa\sigma}\right)^{\frac{\gamma-1}{\gamma}}} \tag{7}$$

(at left: Market impact and order book functions for several values of γ)
Equation (7) implies that $\gamma \leq 1$, besides:

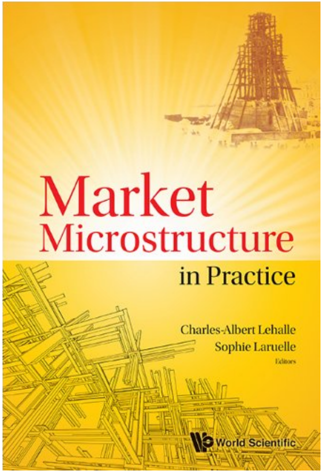
A square root market impact

A square root market impact implies a linear implicit LOB.

With $\gamma = 1/2$, we obtain

$$f_c(S_0 + h) = 2(h - \psi)V$$

Thank You For Your Attention



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